



Math 210 Linear Algebra

Course Description

A first course in vectors, matrices, vector spaces, and linear transformations. The ideas in this course serve as an introduction to more abstract mathematics courses at the junior-senior level, and also covers many useful applications outside mathematics. Topics include: vectors, operations on matrices, inverse of a matrix, solution of systems of linear equations, rank of matrix, vector spaces and subspaces, linear dependence and independence, basis and dimension, linear transformation, sums, composites, inverses of linear transformations, range and kernel of a linear transformation, determinants, eigenvalues and eigenvectors, orthogonality and inner product spaces, and real quadratic forms.

Illinois Articulation Initiative (IAI) number: N/A

Credit and Contact Hours:

| | |
|---------------------|----------|
| Lecture | 3 |
| Lab | 0 |
| Credit Hours | 3 |

Prerequisites: Minimum grade “C” in MATH 170 or equivalent. Students may enroll concurrently in MATH 170.

Books, Supplies, and Supplementary Materials

A. **Required Textbooks**

Cengage Unlimited Subscription. WebAssign will be used for online coursework (homework, quizzes, tests, etc.) and can be accessed by logging into iCampus/Canvas and selecting this course. If you are comfortable reading the textbook on the computer, you may use the eText alone. **There is no need to purchase a physical textbook for this course; the Cengage Unlimited Subscription for the eText and WebAssign was included in your course fees. Registration instructions are posted in our iCampus/Canvas site.**

B. **Other Required Materials**

N/A

C. **Methods of Instruction:**

Lecture, Hybrid, or Online

General Education Student Learning Outcome

1. Quantitative Literacy: Students possess the ability to reason and solve quantitative problems from an array of contexts.

Course Learning Outcomes (CLOs)

1. Solve systems of linear equations using a variety of techniques
2. Explain operations on matrices, invertibility, elementary matrices
3. Demonstrate properties of determinants, including row reduction, to evaluate determinants

4. Identify the dimensions of a variety of vector spaces
5. Determine matrix representations of linear transformations and linear operators
6. Apply the Gram-Schmidt orthonormalization process to find an orthonormal basis for a given basis, subspace, or inner product space
7. Calculate eigenvalues, eigenvectors and eigenspaces of matrices and linear transformations
8. Prove various theorems and properties related to matrices, determinants, vector spaces, inner product spaces, and linear transformations

Lesson Learning Outcomes (LLOs)

1. Define "set" and the related terminology.
2. Define "function" and the related terminology.
3. Explain what is meant by and be able to form the composition of functions.
4. Explain what is meant by a system of equations, a solution of the system, a consistent system, an inconsistent system and a homogeneous system of equations.
5. Define "matrix."
6. Explain what is meant by an " $m \times n$ " matrix, a "square matrix of order n ," the " (i,j) entry" of a matrix and the "main diagonal" of a square matrix.
7. Define and determine the equality of two matrices, the sum of two matrices, the difference of two matrices, the product of two matrices, and the product of the scalar and a matrix.
8. Use summation notation in the definition of matrix multiplication and proof of certain matrix properties.
9. Define "transpose of a matrix" and find the transpose of a given matrix.
10. Make a formal or informal proof of various theorems concerning the above objects and operations.
11. State all of the algebraic properties of matrix operations as discussed in class.
12. Prove selected algebraic properties of matrix operations as well as various theorems which are off shoots of these properties.
13. State what is meant by the zero-matrix, by a diagonal matrix, a scalar matrix, and the identity matrix of order n .
14. Define "upper triangular form" and "lower triangular form" for a matrix.
15. Define "singular" matrix, "nonsingular" matrix, and "inverse" of a matrix and find the inverse of a given matrix when it arrives.
16. Prove various theorems concerning the objects mentioned in Objectives 13 - 15.
17. Explain the connection between singular and nonsingular matrices to the solution of a system of equations.
18. Define " n by n " elementary matrices of type I, II, or III.
19. Prove selected theorems concerning the operation of elementary matrices on a given matrix.
20. Use elementary matrices to develop a technique for finding the inverse of a given matrix.
21. Explain what is meant by row-reduced echelon form for a matrix and transform a given matrix into row-reduced echelon form.
22. Define the three elementary row operations on a matrix.
23. Explain what is meant by one matrix being row equivalent to a second matrix.
24. Prove various theorems concerning row equivalence and row-reduced echelon form.
25. Use matrix techniques discussed in class to solve systems of linear equations.
26. Define "real vector space" and explain the significance of each of the components of the definition.
27. Give examples of a vector space.
28. Define "subspace of a vector space" and give examples.
29. Determine whether or not a given object is a vector space or subspace.
30. Use appropriate notation, work problems, and prove selected theorems involving vector spaces and subspaces.
31. Define "linear combination" of a set of vectors.
32. State what is meant by a set of vectors "spanning" a vector space.
33. Explain what is meant by a linearly dependent or linearly independent set of vectors.

34. Define a "basis" for a vector space.
35. Explain what is meant by a nonzero vector space.
36. Define the dimension of a nonzero vector space.
37. Give examples, use appropriate notation, work problems, and prove selected theorems concerning linear dependence and independence, bases, and dimensions of vector spaces.
38. Define "row space" and "column space" of an m by n matrix.
39. Explain what is meant by the row (column) rank of a matrix.
40. Discuss the structure of a linear system of equations.
41. Define the "determinant" of an n -by- n matrix and evaluate the determinant of a given matrix.
42. Discuss and prove the various properties of determinants and use these properties to aid in solving problems involving determinants.
43. Define the "minor" of an element a_{ij} of a matrix A .
44. Define the "cofactor" of an element a_{ij} of a matrix A .
45. Explain and perform the process of finding a determinant by cofactor expansion.
46. Define the "adjoint" of a matrix A and find the adjoint of a given matrix.
47. Use appropriate notation and prove selected theorems which demonstrate the connection among the inverse of a matrix, the determinant of a matrix, and the adjoint of a matrix.
48. Apply determinants in other selected situations as discussed in class.
49. Define the dot product of two vectors and discuss and/or prove its properties.
50. State the Cauchy-Schwarz inequality and the triangle inequality by cofactor expansion.
51. Define the "distance" between two vectors and what are "orthogonal" vectors.
52. Explain what is meant by an orthogonal set of vectors and an orthonormal set of vectors.
53. Define and calculate the scalar projection and vector projection of one vector on another.
54. Use appropriate notation, work problems, and prove selected theorems concerning inner products, the Cauchy-Schwarz and triangle inequalities, distance and orthogonality.
55. Discuss and use the Gram-Schmidt Process for vectors.
56. Define "linear transformation" of a vector space V into a vector space W .
57. State what is meant by the "null space" and "range" of a linear transformation.
58. Explain what is meant by the matrix representation of a linear transformation.
59. Find the matrix representation of a given linear transformation.
60. Define the "sum," "scalar multiple" and "composition" of linear transformations and thereby define a vector space of linear transformations.
61. State what is meant by a vector space of matrices.
62. Explain the concept of a coordinate vector with respect to an ordered basis.
63. Find how coordinate vectors transform under a change of basis.
64. Define "similar matrices."
65. Give examples, use appropriate notation, work problems, and prove selected theorems concerning rank of a matrix, linear transformations, null spaces, ranges, vector spaces of linear transformations, and vector spaces of matrices.
66. Define "diagonalizable linear transformation" and give example space of matrices.
67. Define "eigenvalue" and "eigenvector" of a linear transformation, give examples, and find the eigenvalues of eigenvectors of a given matrix.
68. State what is meant by the characteristic polynomial of a matrix.
69. Work problems based on the definitions mentioned in objectives 64-68 and theorems based on those definitions.
70. Explain what is meant by a symmetric matrix and skew symmetric matrix, and determine whether or not a given matrix is symmetric or skew symmetric.
71. Discuss and/or prove the theorems connecting diagonalization and symmetric matrices.
72. Define "Real Quadratic Form" and "equivalence" of real quadratic forms.
73. Explain what is meant by congruent matrices.
74. Use appropriate notation, work problems and prove selected theorems involving quadratic forms.

Final Course Grading Scale

| Grade | Percentage |
|-------|----------------|
| A | 90-100% |
| B | 80-89% |
| C | 70-79% |
| D | 60-69% |
| F | lower than 60% |

Faculty Commitment

Faculty members are committed to providing a quality learning experience through thoughtful planning, implementation, and assessment of course activities. They are also committed to being readily available to students throughout the semester by returning e-mails and phone calls within 48 hours and to returning graded course work within a week. Furthermore, they are committed to selecting appropriate course materials and making them available in an organized and timely manner.

Student Commitment

For every credit hour a student is enrolled in, they should expect to spend at least 2 hours outside of class studying, working on assignments, and preparing for class each week of the fifteen-week semester. For example, for this three credit-hour class, students can expect to spend three hours per week in class actively engaged in learning the material by participating in face-to-face classes or viewing lectures and instructional material online. In addition, students should expect to spend another six hours per week outside of class completing homework and assignments, posting to discussion boards online, or studying for quizzes and tests. This means students should spend a minimum of 9 hours per week engaged in achieving the learning outcomes for this course. If you are not achieving your desired results in this class, you should consider increasing your prep time outside of class, in addition to using available resources such as instructor office hours and tutoring services.

By registering for this course, you commit yourself to active participation in course activities as well as the submission of all assignments and exams on time. Furthermore, you commit to accessing the course site and checking your JJC e-mail several times a week.